

What is a Game for Geometry Teaching: Creative, Embodied and Immersive Aspects

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Abstract: Game based Learning (GBL) has been promoted as a way to enhance science, technology, engineering and mathematics education, in several aspects. In this paper we aim at conceptualizing the creative, embodied and immersive potentials in the context of teaching geometry in primary school. We review two cases where the concept of “game” is related to the development of pupils’ creativity and innovation skills. One intervention is inviting pupils to become designers at a “game factory”, by using the digital mathematics tool GeoGebra. The second intervention uses mobile technology to have students participate in a collaborative game requiring them to take part in an embodied activity outside the classroom. In the paper we develop a model that view game based creative learning in as a combination of constructive, immersive, and reflective aspects. We do that by considering different meanings of the word “game” in a mathematics education context. That is game as a medium, game as a framing of educational processes, and games as an object. Considering games as media highlights the similarities with texts and any other means of delivering content. We can ask what message a specific game conveys, and discuss how well suited the game is compared to other mediations of the same content. Games can be described as “se-miotic domains” that allow players to interact with knowledge and make sense of the world (Gee, 2003). Games can act as a framing used to govern and plan educational processes, either as direct motivational driver, aiming at engaging more students in certain planned activities (Hamari, Koivisto, & Sarsa, 2014), or as an established form of process control in which complex situations can be played out. In a game the player act according to rules and the process has a natural direction towards finishing or advancing in the game. Furthermore games can frame educational processes by challenge the learners perspective through narratives and role-playing (Shaffer, 2006). Pupils creative design competencies and motivation to create and develop games have been documented as unusually relevant and high (Kafai, 1995; Tekinbas, Gresalfi, Peppler, & Santo, 2014). Mathematical thinking s can be used to think game scenarios through, to govern the competitive aspects of a game and ensure that the gameplay is fair and balanced in terms of the involved struggle. Game literacy, both in terms of playing, producing and discussing games, does thus relate to mathematical literacy and knowledge.

Keywords: serious games, embodiment, epistemic games

1. Introduction: Game oriented learning and STEM

The use of educational games is a promising path to enhance science, technology, engineering and mathematics (STEM) teaching. It is argued that such games can motivate, engage, provide students with relevant educational experiences more in sync with the competences needed in society and allow for better learning process. However many of the educational products that builds on game mechanics are often criticized for being mere entertainment in the sense that they combine an entertaining gameplay with simple root learning tasks providing classical curricular content. Hence the relation between games and STEM education is richer than providing a competition and narrative around drill and practice activities or immersing students into role playing activities.

The main problem that we address in this paper is how we should understand the many possible relations between games and STEM education. We challenge the naïve idea that games is a (perhaps very good) substitute for genuine teaching, and we use two cases of game related activities in the area of geometry to develop a conceptualization of the ways that games relate to STEM education, with a special focus on embodiment and innovation. Hence we will attempt to approach game based learning in a much broader fashion than games as instrumental learning machines. Simultaneously we aim at focusing on specific aspects of learning (creativity and embodiment) and specific topics (geometry), in order to develop a situated understanding of what game oriented learning can be. After a brief state of the art on games, mathematics, embodiment, and creativity, we present two cases of geometry teaching where the relation between embodiment, creativity and immersion are different. Following this case description we develop and discuss a model for game oriented STEM learning distinguishing the various roles of game as medium, framing, and object.

2. Games, mathematics and creativity

Computer related activities that enhance collaboration and learning are often referred to as computer supported collaborative learning activities (Koschmann 1996). In this tradition Shaffer (2006) has developed the

concept *epistemic frame* to describe how participants in a practice develop shared ways to view the world. Taking a sociocultural approach this framework distinguishes between skills, knowledge, identities and values as pragmatic categories that describe an epistemic domain. Furthermore this framework introduces the notion of epistemic game as a “*game that deliberately creates the epistemic frame of a socially valued community by re-creating the process by which individuals develop the skills, knowledge, identities, values and epistemology of that community*” (Shaffer, 2006). Shaffer’s framework combines investigation and understanding of practice and design of learning material (epistemic games) that uses this practice to formulate learning.

The van Hiele’s have made an extensive work on understanding the nature of geometrical thinking (van Hiele’s cited in Clemments; 2003). They distinguish four levels of geometrical understanding 1) Description (being able to distinguish and somehow talk about shapes), 2) analysis (being able to distinguish shapes according to defining properties), 3) Abstraction and 4) Proof. This framework suggests that a profound knowledge on the lower stages (1 and 2), is a necessary prerequisite for developing the higher levels of geometric thinking.

Kress and van Leeuwen (2001) define multimodality as “the use of several semiotic modes in the design of a semiotic product or event, together with the particular way in which these modes are combined.” In traditional teaching there has often been a dominance of writing, and verbal communication, but the use of new media in learning makes it possible to integrate other modalities in the design of learning, and explore their potentials for engaging and motivating the learners. Other forms of knowledge are predicated on the representations and modes, enabling learning in different ways, that may be relevant to the learner within different frames, as Kress suggests (2009) “there is no meaning without framing”.

The embodied nature of mathematical thinking and the formation of mathematical concepts have been investigated and discussed thoroughly in the mathematics education community, and among philosophers of mathematics. Some philosophers and cognitive scientists argue that mathematics is in fact an output of human cognition and the way humans with bodies act in and experience the world (Lakoff and Nunes, 2000). Yet others consider this analysis superficial because it does not reflect the extent to which mathematical activity demands deliberate cultivation of analytical thinking styles, and hence are very different from what could be considered a natural mode of thinking (Leron & Hazzan 2006). Disregarding this controversy there is a consensus that experience with mathematical phenomenon’s does have some value in mathematical concept formation.

From a didactical/educational point of view this philosophical controversy is not so much over whether or not bodily activities are relevant in mathematics education, as it is a discussion about whether or not such bodily activities should always be complemented by an analytical approach.

Developing mathematical creativity with students is an important ambition in mathematics education, however mathematical creativity can be considered as *with* or *within* mathematics, meaning either creativity as using mathematics as tool to express oneself, or developing solutions valued as creative within the field of mathematics as a school discipline or scientific discipline. And furthermore two basic metaphors exist for mathematical creativity, namely construction and problem solving (Misfeldt, forthcoming).

3. Case one: Creative digital mathematics

The research and development project “Creative Digital Mathematics” (Misfeldt & Zacho, forthcoming) aims to develop and implement a new approach to the introduction of digital mathematical tools to introductory and middle schools students. The research explores potentials in using the children’s creative competencies as a vehicle for teaching skills with mathematical tools. Furthermore the intervention challenges the students’ conception of teaching and learning of mathematics, towards a more free and student driven situation. The project follows the ideas of Papert (1980) as well as the ideas in game based learning titled “scenario based education” (Hanhøj 2011, Misfeldt 2015). In the intervention we aim to develop mathematical learning environments that allows pupils’ to work with digital media and mathematical representation allowing them to appropriate GeoGebra to their own need.

The project has been running from March 2011 to June 2014, and been through four cycles of design and intervention ((1) one grade 5 class, (2) two grade 3 classes, (3) 40 primary school teachers and (4) 40 secondary school teachers). In each intervention, students have developed board games or other visual structures using the tool GeoGebra (see Hohenwarter & Jones (2007) for a description of the tool).

The pupils' work is organized by a simple web based interface, and starts with a few simple drawing tasks, continues to solve a number of mathematical tasks before they start developing their project.

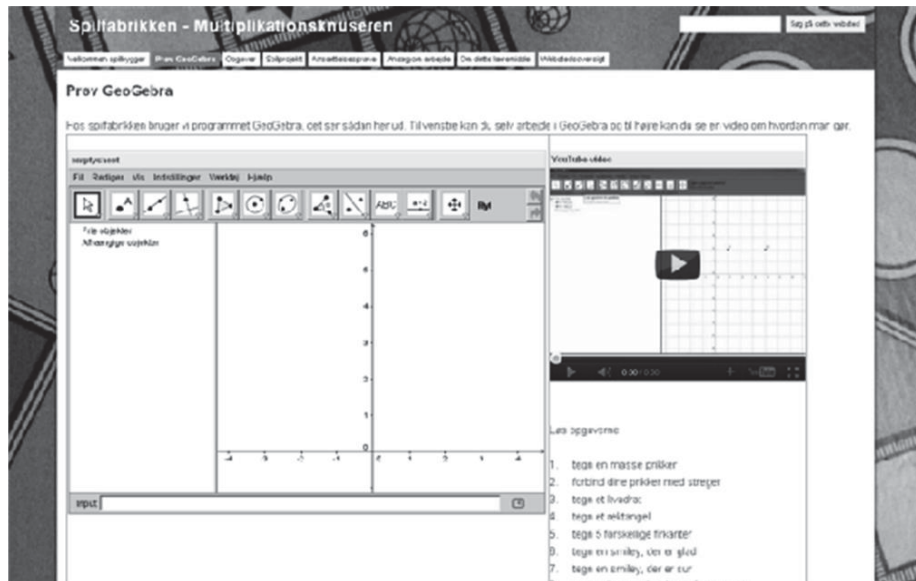


Figure 1: Screenshot from “multiplication crusher” with tasks (text field, bottom left), video-introduction (top left) and an embedded GeoGebra workspace

The data we have consist of the teaching materials developed by the teachers, together with the material, the teachers also provided a description of learning goals and a detailed description of their intentions with the material, student products, observations of 50 lessons, and interviews with eight students (see Misfeldt & Zacho (forthcoming) for a description of how this material is analyzed),

The pupils were able to take control over the software and appropriate it to their needs. Given that this intervention was the first meeting with the GeoGebra for most pupils, the age of the pupils (primary and middle school grade 1- 6), and the time used with the project (approximately ten lessons), we do consider this a good result.

In both design/intervention cycles all pupils developed a game. Most, but not all, games had some mathematical theme. Designing board games was accepted as a meaningful activity by almost all children.

The interviews with pupils (Rosenkvist, 2012) revealed that the pupils considered their work as mathematical work. The pupils in general felt that the mathematics classes were much freer, building more on their own ideas.

Through the interventions we have realized that there is a strong benefit from structuring the students' work with a digital learning environment, both for the pupils whose learning is structured, but also for the teachers who are able to articulate their ideas about digital mathematics teaching in a collaborative way..In that sense the project uses the teachers' competencies as didactical designers as a vehicle to introduce ICT in mathematics teaching. Since the teachers take part in the design of the learning environment the project attempts to capitalize on teachers' creativity in order to change teaching and learning of mathematics towards including digital tools.

4. Case two: Mobile learning environments

The research and development project “Mobile Learning Environments” has explored the use of a mobile game prototype “MathX: The Search for Ancient Wisdom” in the teaching of mathematics for students age 14-16 in the Nordic schools. The project has developed a game for learning mathematics in secondary school, and subsequently implemented it on 3 different mobile platforms, which have been tested with learners in different educational settings. The platforms range from a conventional Mobile phone to Smartphone, to a Sensor-based platform.

The project was conducted in collaboration between Nordic partners from Universities as well as industry partners, like Nokia. The research involved user-tests done on location in classrooms and outdoor areas in Denmark, Sweden and Finland where the game was used as an intervention in the math-class. The research design used a mixed-methods action-research approach, and included pre- and posttest questionnaires, as well as video-observations and interviews with students and teachers involving three schools in the three Nordic countries. The study involved a class in each of the schools with students grades seven to nine. The material was subsequently analyzed in order to understand the affordances of a mobile game-based learning environment in relation to the learners' experience of engagement, motivation and their modal preferences for learning. The research method allowed for triangulation of the data from the surveys, observations and interviews, thereby strengthening the validity of the findings.

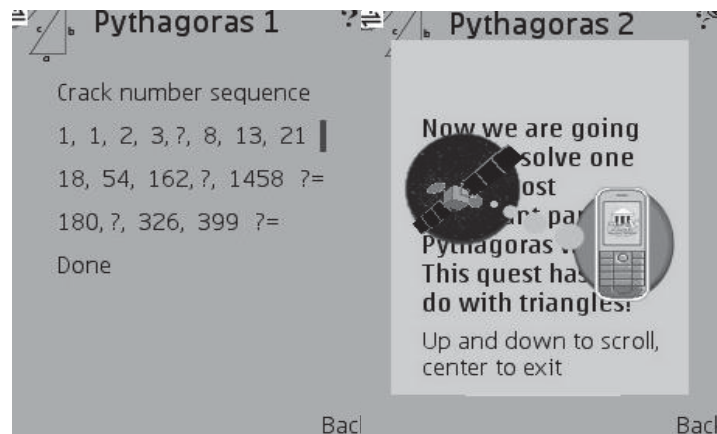


Figure 2: Screen-shots from the mobile game on Pythagoras, where tasks had to be solved in order to move forward in the environment

The mobile game and the narrative about Pythagoras in which it was embedded as a quest for solutions to problems, provided an experiential framework for solving tasks that were contextually anchored in the physical environment and added a potentially affective dimension to the learning situation. The teachers also reported this experience of the learners' engagement as a positive outcome of using this mobile game-based approach. This "fun-factor", which some of the students reported made learning math into something much more enjoyable and tangible than they had ever experienced before, according to the qualitative data and the data from the questionnaires.



Figure 3: A collaborative learning situation in the gameplay

The design of the mobile game prototype “MathX: The Search for Ancient Wisdom” has implemented theories of collaborative learning and is supposed to be played in groups of three to four students, depending on the mobile device used. The results from the questionnaires and the qualitative interviews after the game play tests, showed that the experiences with the game led to a more positive attitude towards collaboration, and that the majority of the students perceived themselves as fully involved in the work of the group. Field observations, however, showed that not all learners were involved in the actual problem solving in the group, something which the game-design in a further development should seek to scaffold through developing user roles that offer adequate challenges to all players.

From the observations and interviews in relation to the testing it was obvious that the majority of the learners enjoyed the experience of working in a mobile game-based learning environment. The analysis of the data from all three countries showed that the affordances which the game-based mobile learning environments were providing the learners were related to:

- The collaborative learning which provided a setting for social interactions
- The multiple multimodal representations
- The game-embedded learning environment
- The context sensitivity
- The portability of the devices supporting learning situations outside the classroom
- Framing of physical activities, that allowed for measuring, and moving about

These affordances are important factors to consider when developing or evaluating math in game-based mobile learning environments.

In the previous sections we have shown examples of game related educational activities that are different from sugar coated drill and practice exercises. We will use these examples as an outset of a discussion of the possible roles of games in teaching and learning of mathematics. This discussion is organized around two main themes; (1) understandings of what games are and can be considered in mathematics education (as medium, framing and object), and (2) theoretical conceptions of the learning potentials in games.

The resulting picture reveals a broad scope of educational possibilities related to the uses of games in mathematics teaching. The result of the analysis can be seen as a first attempt to untangle this range of possibilities.

5. What is a game in mathematics teaching and learning

In the following discussion we take three different approaches, considering games as a medium for delivering content, as a framing that governs and communicates a certain process, and as an object for students and teachers to develop, discuss and analyse.

5.1 Game as medium

Considering a game as a medium highlights the similarities with texts and any other means of delivering content. This is perhaps the most mainstream and important way to look at the educational potentials of games. We can ask what message a specific game conveys, and discuss how well suited the game is compared to other media-tions of the same content. James Paul Gee (2003) describes games as “semiotic domains” that allow players to interact with knowledge and make sense of the world. In Gee’s educational vision games in general and video-games in particular are considered media that the player “reads/consumes”. Since a game is often read/played in a far more interactive way than texts in other media, there are specific learning potentials related to games. This idea is followed in Devlin’s description games as a world where all sorts of mathematical phenomenon’s can be conveyed.

In both of interventions described we can consider the involved games as media. In the creative digital mathematics project students develop games conveying knowledge about multiplication, and they work in a game like on-line learning environment designed to convey knowledge about multiplication and the uses of the digital tool GeoGebra. In the mobile learning game the content of the game is even more explicit, as the game had a story-line about Pythagoras, which connected to the tasks that had to be solved to advance in the game. The game

was multi-modally (Kress 2003) mediated through different mobile devices and tangible it-artefacts which allowed for learning through haptic feed-back to the solving of mathematical problems linked to Pythagoras and the game -play was supported by audio and visual storytelling on the mobile devices.

The mobile game then functioned as a medium for multi-modal learning, enabling the learners to engage with mathematical learning drawing on their literacy about the multi-modal game medium. It is clear that a view of games as media for delivering content does not take all aspects of the game medium into account – especially the interactivity and feedback possibilities. These interactive possibilities are a natural aspect of games considered as media as described by Gee (2003).

5.2 Game as framing of educational processes

Apart from games being a medium with some specific interactive, motivational and mathematical aspects, games can also be used to govern educational processes. Games can be used as a mean of controlling and driving complex educational activities. This didactical potential has several aspects. Games can be used as a direct motivational driver, aiming at engaging more students in certain planned activities. Furthermore games are an established form of process control in which complex situations can be played out. In a game the player act according to rules and the process has a natural direction towards finishing or advancing in the game (even though students also might just wander off exploring the environment). Finally games can challenge the learners' perspective through narratives and role-playing. This potential been investigated by David W. Shaffer (2006), and according to him games allows students to act through an epistemic starting point different from their student perspective.

The learning potentials of students designing games for each other has been looked at in a study by Gjedde, Horn, Sørensen (2011) exploring the use of students design of a learning game for energy awareness. It found that it framed the students' reflection on a content level as well as at an aesthetic level, and that the process of designing the game called for a greater level of cognitive involvement than playing the game.

In the creative digital mathematics project students engage in an epistemic role playing game, and in that sense they act as designers rather than students of mathematics. This reframing allows students to develop products (board games) that are not mathematics per se, but does activate and draw on mathematical skills and competences.

In the Mobile Learning Environment the framing potential is even more explicit. The advances made in the game allow students to engage in curricular activities while they simultaneously are participating in a location-based game. Without the gameplay the motivation for and direction of the educational process would be hard to maintain, and the process would be difficult to control and facilitate for the teacher.

According to Klopfer (2008) mobile games can be used to tackle real world problems in a playful environment, a frame in which the situated complex and dynamic problem-solving challenges and motivates the learner to develop scientific argumentation.

5.3 Game as an object

Apart from delivering content and governing classroom processes games are objects themselves. This means that games can be used in education as object for analysis, as something students develop and even as an educational deliverable. In the project "Creative Digital Mathematics" we use the pupils creative design competencies and motivation to create and develop games (previously investigated by Kafai (1998)). These games are products that have a height level of mathematics in them. Mathematics can be used to think game scenarios through, to govern the competitive aspects of a game and ensure that the gameplay is fair and balanced in terms of the involved struggle. Game literacy, both in terms of playing, producing and discussing games, does thus relate to mathematical literacy and knowledge, and it is important to include this aspect in the curriculum and learning designs.

6. Learning potentials

In the following we will focus on some of the potentials that games have for providing a learning environment that includes creativity, embodied cognition and collaboration.

6.1 Bridging mathematics and creativity

In the Creative digital mathematics project we use the potential of games as an object to ask students to make games, and hence to work in a creative way and use mathematical concepts. In order for the pupils to view themselves as working in a creative way, we also use the potential of (role playing-) games as a framing to allow the children to participate in a different practice inspired by professional design activities

Games have the potential of reaching between the realm of human experience and aesthetics on one side and numbers, calculations, balance, and geometric shapes on the other side. Working with games, pupils can activate several modes of thinking in their creative process and also get a feel for the real life applications of mathematical competencies that are linked to a very engaging and creative learning experience.

6.2 Activation of embodied cognition

Playing games does often combine the kinesthetic dimension with the cognitive and this makes the game format well suited for framing processes that enhance the embodied nature of mathematical thinking. In the mobile learning game the participants are explicitly pushed towards exploring mathematical problems with their bodies. We argue that the game framing adds meaning and recognition to these activities. The van Hiele theory of geometrical thinking shows the importance of grounding higher conceptual thinking in very concrete tactile conceptions of the geometrical shapes. Games can play an active role in that process. Firstly games allow the teaching and learning of geometry to be more spatial and embodied by bringing the students out of the classroom as seen in the Mobile Learning Environment project. This process has the potential of providing the necessary grounding of the geometrical concepts. In the Creative Digital Mathematical project the children used mathematical software to develop a visual and interactional project. In order to make the visual shapes that the students care for they need to refer to the definitions (for instance by creating a circle using center and radius. This process has the potential of moving up the van Hiele levels towards the analysis and abstraction level.

6.3 Enhancing collaboration

Games are often connected to collaboration, where players are collaborating in teams in order to solve problems and advance in the gameplay. Teams and opponents are a natural aspect of playing games, and hence framing an activity as a game can constitute these roles. Furthermore pupils development of games as (partly mathematical) objects, can be a way of ensuring that students with different interests can have something meaningful to collaborate on.

Games can be a vehicle for Problem- Based Learning (Savery & Duffy 1996) ,which is an approach to learning with a focus on learning through the solving of relevant problems, rather than through a presentation. In relation to this games can be understood as frames for the collaborative problem-solving that anchors the problems in a fictional setting that can be very engaging for the learners.

Table 1: The three conceptions of game in geometry teaching

	Bridging mathematics and creativity	Activation of embodied cognition	Enhancing collaboration
Medium	The expression through the medium of game calls for creativity as well as mathematical competencies	The medium can combine the kinesthetic dimension with the cognitive	Games can be a medium for collaboration through the gameplay
Object	Games as objects serve as bridge between mathematics and creativity		Games can be the objects of shared reflection in a group of learners
Framing	Epistemic games can frame creative problem solving	The game can frame activities of embodied cognition	The shared framework supports collaborative workflow by the learners

7. Conclusion

In this chapter we have explored two cases where a combination of Game Based Learning with a creative, collaborative and embodied pedagogy has presented an alternative approach to the teaching and learning of mathematics. We have developed a conceptualization of how games can be conceptualized in STEM learning situations, as either; medium, frame or object. Furthermore we have distinguished learning potentials in relation to these ways of thinking about games.

We have pointed to some approaches that may frame learning designs that enhance collaboration in the classroom while bridging mathematics and creativity.

In order to explore the learning potentials of this approach further, more research should be done focusing on the process of the learners' game-creation as well as the parameters for the learning design of mobile games in education.

References

- Brown, Collins and Duguid (1989) Situated cognition and the culture of learning, *Educational Researcher*; v18 n1, pp. 32-42, Jan-Feb 1989.
- Bishop, A. J. (2008). The role of games in mathematics education, in *The Game as Teaching Strategy*, eds Fransesc Lopez Rodriguez, Editorial Laboratorio Educativo, Venezuela, pp. 23-34.
- Brousseau, G. *Theory of didactical situations in mathematics: : Didactique des mathématiques, 1970-1990*. Dordrecht (Holland): Kluwer Academic Publishers.
- Bruner, J. (1985). Vygotsky: An historical and conceptual perspective. *Culture, communication, and cognition: Vygotskian perspectives*, 21-34. London: Cambridge University Press.
- Clemments, D. H. (2003) *Teaching and Learning Geometry* in Kilpatrick, J., Martin, W. G., Schifter, D., & National Council of Teachers of Mathematics. (2003). *A research companion to principles and standards for school mathematics*. Reston, VA: National Council of Teachers of Mathematics.
- Devlin, K. J. (2011). *Mathematics education for a new era: Video games as a medium for learning*. Natick, Mass: A K Peters.
- Gee, J. P. (2003). *What video games have to teach us about learning and literacy*. New York: Palgrave Macmillan.
- Gjedde, L. (2009) Affordances of Mobile Learning Environments . DPU/Aarhus University.
- Gjedde, L. Horn, F. Sørensen H. (2011) *Innovativ Energiundervisning*. DPU/Aarhus University.
- Hanghøj, T. (2011). Clashing and emerging genres: the interplay of knowledge forms in educational gaming. *Designs for Learning*, 4(1), 22.
- Hatch, T. and H. Gardner 'Finding cognition in the classroom: an expanded view of human intelligence' in G. Salomon (ed.) *Distributed Cognitions. Psychological and educational considerations*, Cambridge: Cambridge University Press. (1993)
- Hohenwarter, M., & Jones, K. (2007). Ways of linking geometry and algebra: the case of GeoGebra. In D. Küchemann (Ed.), *Proceedings of the British Society for Research into Learning Mathematics*. 27(3), University of Northampton, UK: BSRLM
- Kafai, Y. B., Franke, M. L., Ching, C. C., & Shih, J. C. (1998). Game Design as an Interactive Learning Environment for Fostering Students' and Teachers' Mathematical Inquiry. *International Journal of Computers for Mathematical Learning*, 3, 2, 149-184.
- Koschmann, T. D. (1996). *CSCL, theory and practice of an emerging paradigm*. Mahwah, N.J: L. Erlbaum Associates.
- Klopfer, E. (2008). *Augmented learning: Research and design of mobile educational games*. Cambridge, Mass: MIT Press
- Kress, G.R. (2003). *Literacy in the new media age*. London: RoutledgeFalmer
- Kress, G.R. and van Leeuwen, T. (1996). *Reading Images: the grammar of graphic design*. London: Routledge
- Kress, G.R. and Van Leeuwen, T. (2002). *Multimodal Discourse: the modes and media of contemporary communication*. London: Edward Arnold
- Kress, G.R. (2009) *Multimodality*. London: Routledge
- Lakoff, G., & Núñez, R. E. (2000). *Where mathematics comes from: How the embodied mind brings mathematics into being*. New York, NY: Basic Books.
- Lave, J and Wenger, E *Situated Learning: Legitimate Peripheral Participation*. Cambridge: University of Cambridge Press. (1991).
- Leron, U., & Hazzan, O. (2006). The Rationality Debate: Application of Cognitive Psychology to Mathematics Education. *Educational Studies in Mathematics*, 62, 2, 105-126.
- Misfeldt, M. (forthcoming). Making Meaning of Creativity and Mathematics Teaching, Proceedings of NORMA 14 the seventh Nordic conference on mathematics education, Turku, June 2014.
- Misfeldt, M. (2015). Scenario Based Education as a Framework for Understanding Students Engagement and Learning in a Project Management Simulation Game. *Electronic Journal of E-Learning*, 13(3), 181-191.
- Misfeldt, M. & Zachø, L. (forthcoming) Scenario Design as a Way of Introducing Technology in the Primary Level Mathematics Classroom: appropriation of GeoGebra and game, as resources for supporting teacher collaboration, *Journal of Mathematics Teacher Education*, New York: Springer

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- Papert, S. (1980). *Mindstorms: Children, computers, and powerful ideas*. New York: Basic Books.
- Prensky, M. (2001). *Digital game-based learning*. New York: McGraw-Hill.
- Rosenkvist, E. (2012) *Elevers oplevelse af matematik med GeoGebra under frie undervisningsrammer*. Master Thesis Aarhus University.
- Savery J. and Duffy, T. 1996. Problem based learning: An instructional model and its constructivist framework", in B. Wilson (Ed.), *Constructivist learning environments: Case studies in instructional design*, Educational Technology Publications, Englewood Cliffs, NJ,USA, 1996, pp. 135-148.
- Shaffer, D. W. (2006). *How computer games help children learn*. New York: Palgrave Macmillan.
- Sinclair, N., Healy, L., & Sales, C. O. (January 01, 2009). Time for telling stories: narrative thinking with dynamic geometry. *Zdm*, Berlin: Springer, 41, 4, 441-452.
- Vygotsky, L. (1978). *Mind in society: The development of higher psychological processes*. Cambridge: Harvard University Press